DBI inflation

- A natural inflation model that can generate large equilateral NG   Silverstein & Tong '03
- Inflaton is identified as a position modulus of a probe brane in extra-dimesnions

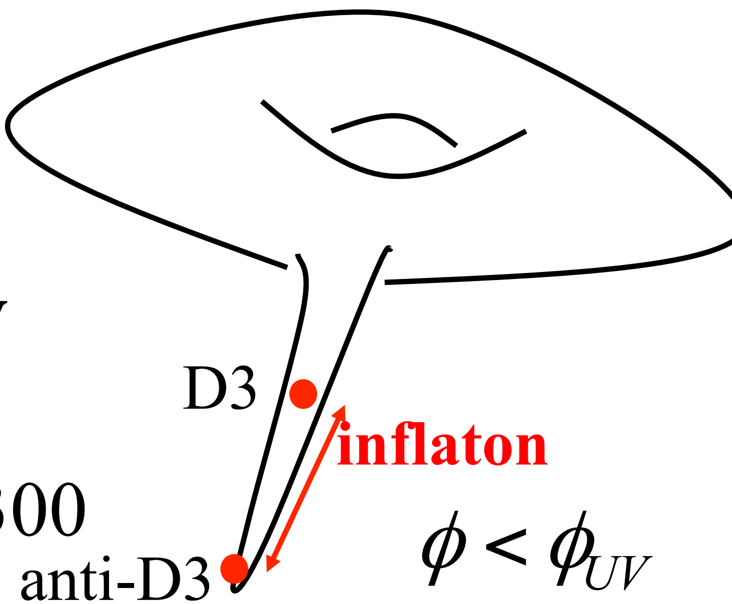
A small sound speed enhances NG

$$f_{NL}^{equi} = -\frac{35}{108}\frac{1}{c_s^2}$$

- Single field models in string theory are severely constrained

$$r = 16 c_s \varepsilon < 10^{-7} \quad \Rightarrow \quad f_{NL}^{\rm equil} > 300$$

$$1 - n_s \sim 4\varepsilon \sim 0.04 \pm 0.013$$



Huston et.al '07, Bean et.al. '07

Multi-field DBI inflation

- DBI inflation is naturally multi-field (i.e. 6 extra-dimensions = 6 fields)

  Renaux-Petel, et.al '08, '09, Arroja, Mizuno Koyama '08

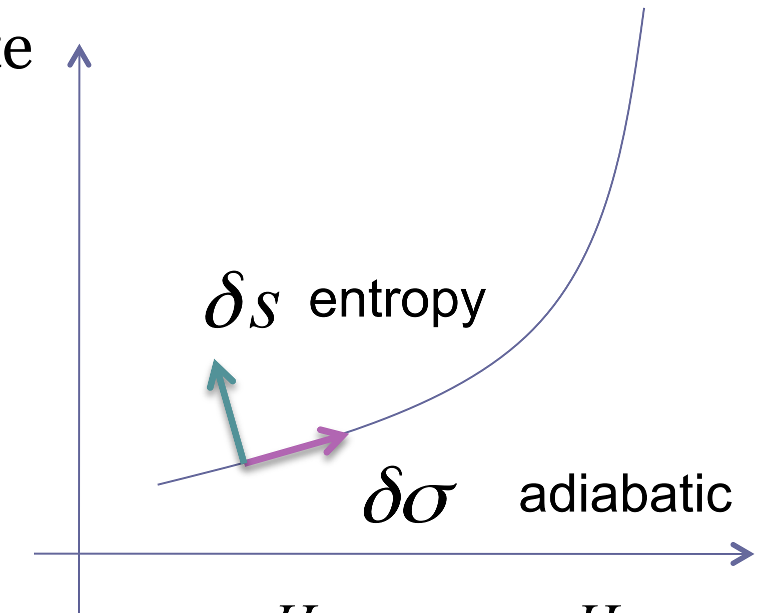
- Multi-field effects could ameliorate the problem $\zeta = \zeta_* + T_{RS} S_*$

$$r = 16 \varepsilon c_s \frac{1}{1 + T_{RS}^2}$$

$$f_{NL}^{equi} = -\frac{35}{108} \frac{1}{c_s^2} \frac{1}{1 + T_{RS}^2}$$

- Trispectrum is easier to detect

$$\tau_{NL}^{equi} \propto T_{RS}{}^2 f_{NL}^{equi\,2}$$



$$\zeta = \frac{H}{\dot{\sigma}} \delta\sigma, \quad S = \frac{H}{\dot{\sigma}} \delta s$$

Arroja, Mizuno Koyama 'Tanaka '09, Mizuno Arroja Koyama '09, Renaux-Petel '09

DBI Galileons

- Single field extension including higher order derivative interactions
- A probe brane in 5D

de Rham & Tolley '10, Burrage et.al. '11, Goon et.al.'11, Mizuno Koyama '10, …

A general brane action that gives the 2nd order e.o.m

$$S = \int d^4x\sqrt{-h}\left(\alpha_1 + \underbrace{\alpha_2 K + \alpha_3 R[h] + \alpha_4 \mathrm{K}_{GB}}_{\text{Relativistic Galileon terms}}\right)$$

DBI — Relativistic Galileon terms

$$h_{\mu\nu} = g_{\mu\nu} + \partial_\mu\phi\partial_\nu\phi$$

$$K_{\mu\nu} = -\gamma\,\partial_\mu\partial_\nu\phi$$

$$\gamma^{-1} = c_s^2 = 1 + \left(\partial_\mu\phi\right)^2$$

K	${}^{(5)}R[g^{(5)}]$
K_{GB}	${}^{(5)}R_{GB}[g^{(5)}]$
$R[h]$	

Multi-field DBI Galileons

- In higher-codimensions n>1, it becomes multi-field model and for even n>3, the only possible term is the induced gravity term

$$S = \int d^4x \sqrt{-h}\left(-\alpha_1 + \alpha_2 R[h]\right) \qquad h_{\mu\nu} = \eta_{\mu\nu} + \partial_\mu \phi^I \partial_\nu \phi_I \,,\ I = 1, 2, ..$$

Hinterbichler et.al. '10

- In non-relativistic limit $(\partial\phi)^2 << 1$

$$S = \int d^4x \left(-\alpha_1 \frac{1}{2} \partial_\mu \phi^I \partial^\mu \phi_I + \alpha_2 \partial_\mu \phi^I \partial_\nu \phi^J (\partial_\lambda \partial^\mu \phi_J \partial^\lambda \partial^\nu \phi_I - \partial^\mu \partial^\nu \phi_I \Box \phi_J) \right)$$

Padilla et.al. '10 '11

Model

Renaux-Petel, Mizuno, Koyama '11

- Embed a brane in 10-dimensions

$$ds^2 = g^{(n)}_{ab} dX^a dX^b = h^{-1/2}(y^I) g_{\mu\nu} dx^\mu dx^\nu + h^{1/2}(y^I) G_{IJ} dy^I dy^J$$

$$y^I = \frac{\phi^I(x^\mu)}{\sqrt{T_3}}, \quad x^\mu = x^\mu$$

- Induced metric

$$\begin{aligned} \gamma_{\mu\nu} &= g^{(n)}_{ab} \frac{\partial X^a}{\partial x^\mu} \frac{\partial X^b}{\partial x^\nu} \\ &= h(\phi^I)^{-1/2} q_{\mu\nu}, \quad q_{\mu\nu} = g_{\mu\nu} + f(\phi^I) G_{IJ}(\phi^I) \partial_\mu \phi^I \partial_\nu \phi^J \quad f = \frac{h}{T_3} \end{aligned}$$

- Action

$$S = \int d^4x \left[\frac{M_P^2}{2} \sqrt{-g} R[g] + \frac{M^2}{2} \sqrt{-\gamma} R[\gamma] - T_3 \sqrt{-\gamma} - \sqrt{-g} \left(V - \frac{1}{f} \right) \right]$$

Equations of motion

$$M_P^2 G^{\mu\nu}[g] + M^2 \frac{\sqrt{\mathcal{D}}}{h^{3/2}} G^{\mu\nu}[\gamma] = -\frac{1}{f}\sqrt{\mathcal{D}} q^{\mu\nu} - g^{\mu\nu}\left(V - \frac{1}{f}\right)$$

$$\left(\frac{M^2}{h^{3/2}} G^{\mu\nu}[\gamma] + \frac{1}{f} q^{\mu\nu}\right)\left(\delta^I_J + 2fAX^I_J + 4fX^{IK}A_{KL}X^L_J\right)\left(\hat{\Pi}^J_{\mu\nu} + \frac{f_{,}{}^J}{4f}\frac{q_{\mu\nu}}{f}\right) - \frac{G^{IJ}}{f\sqrt{\mathcal{D}}}\left(V_{,J} + \frac{f_{,J}}{f^2}\right) = 0$$

$$q^{\mu\nu} = Ag^{\mu\nu} - A_{IJ}\nabla^\mu\phi^I\nabla^\nu\phi^J\,,$$

$$R_{\mu\nu}[\gamma] = R_{\mu\nu}[q] + \frac{1}{2}\nabla^q_\mu\nabla^q_\nu \ln h + \frac{1}{4}q_{\mu\nu}q^{\alpha\beta}\nabla^q_\alpha\nabla^q_\beta \ln h + \frac{1}{8}(\nabla^q_\mu \ln h)(\nabla^q_\nu \ln h) - \frac{1}{8}q_{\mu\nu}q^{\alpha\beta}(\nabla^q_\alpha \ln h)(\nabla^q_\beta \ln h)$$

$$\begin{aligned} R_{\alpha\beta}[q] &= AR_{\alpha\beta}[g] - A_{IJ}\nabla^\nu\phi^I\nabla^\lambda\phi^J R_{\lambda\alpha\nu\beta}[g] \\ &+ AH_{AB}\left([\hat{\Pi}^A]\hat{\Pi}^B_{\alpha\beta} - (\hat{\Pi}^A\cdot\hat{\Pi}^B)_{\alpha\beta}\right) \\ &+ A_{IJ}H_{AB}\left((\partial\phi^I\cdot\hat{\Pi}^A)_\alpha(\partial\phi^J\cdot\hat{\Pi}^B)_\beta - (\partial\phi^I\cdot\hat{\Pi}^A\cdot\partial\phi^J)\hat{\Pi}^B_{\alpha\beta}\right) \\ &- 2(AX^{LJ} + 2X^{LM}X^{JN}A_{MN})\hat{R}_{LIJK}\nabla_\beta\phi^K\nabla_\alpha\phi^I\,, \end{aligned}$$

$$\hat{\Pi}^I_{\mu\nu} = \Pi^I_{\mu\nu} + Z^I_{AB}\nabla_\mu\phi^A\nabla_\nu\phi^B\,,$$

$$\Pi^I_{\mu\nu} = \nabla_\mu\nabla_\nu\phi^I + \Gamma^I_{AB}\nabla_\mu\phi^A\nabla_\nu\phi^B$$

$$X^{IJ} \equiv -\frac{1}{2}\partial_\mu\phi^I\partial^\mu\phi^J \qquad \mathcal{D} = 1 - 2fX^I_I + 4f^2X^{[I}_I X^{J]}_J - 8f^3X^{[I}_I X^J_J X^{K]}_K + 16f^4X^{[I}_I X^J_J X^K_K X^{L]}_L$$

Background

- Friedman equation

$$3H^2M_P^2 + \frac{3M^2}{c_D^3 h^{1/2}}\left(H - \frac{\dot{f}}{4f}\right)^2 = V + \frac{1}{f}\left(\frac{1}{c_D} - 1\right) \qquad c_D^2 \equiv 1 - f\dot{\sigma}^2$$

$$-\dot{H}\left(M_P^2 + \frac{M^2}{c_D h^{1/2}}\right) = \frac{\dot{\sigma}^2}{2c_D} \qquad \dot{\sigma} \equiv \sqrt{G_{IJ}\dot{\phi}^I\dot{\phi}^J}$$

$$-\frac{M^2}{c_D h^{1/2}}\left(\frac{3}{2}\left(\frac{1}{c_D^2} - 1\right)\left(H - \frac{\dot{f}}{4f}\right)^2 + \right.$$

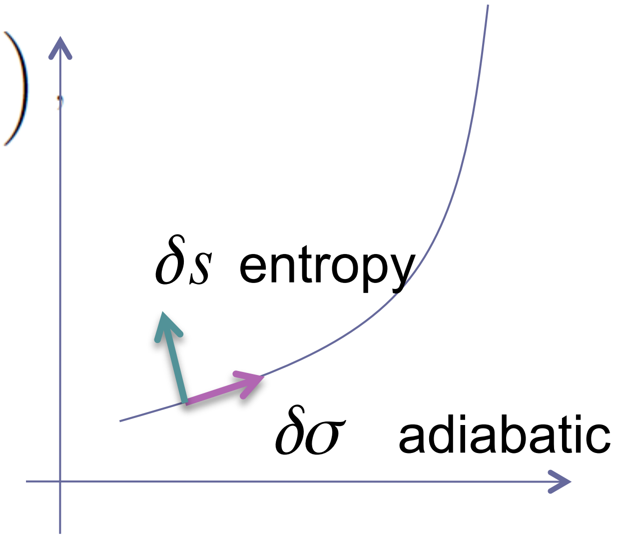
- In the relativistic regime, induced gravity gives a term that breaks the null energy condition

- Conditions for inflation $c_D fV >> 1, \quad \frac{M^2}{M_p^2 c_D^3 h^{1/2}} << 1$

Linear perturbations

Second-order action $c_D^2 \equiv 1 - f\dot{\sigma}^2$ $\alpha = \dfrac{fH^2M^2}{c_D^2 h^{1/2}}$

$$S_{(2)} = \frac{1}{2}\int \mathrm{d}t\,\mathrm{d}^3x\,a^3\left(\frac{\dot{Q}_\sigma^2}{c_D^3}\left(1-3\alpha(3-2c_D^2)\right) - \frac{(\partial Q_\sigma)^2}{c_D a^2}\left(1-\alpha(5-2c_D^2)\right) + \frac{1-3\alpha}{c_D}\left(\dot{Q}_{se}^2 - c_D^2\frac{(\partial Q_{se})^2}{a^2}\right)\right)$$



Avoid instabilities

$$\alpha < \frac{1}{9} \quad (c_D << 1)$$

Sound speeds

$$c_a = \frac{1-5\alpha}{1-9\alpha}c_D, \qquad c_e = c_D << 1$$

Entropic sound speeds are always smaller than adiabatic one and they become the same in the DBI inflation $c_e \le c_a$

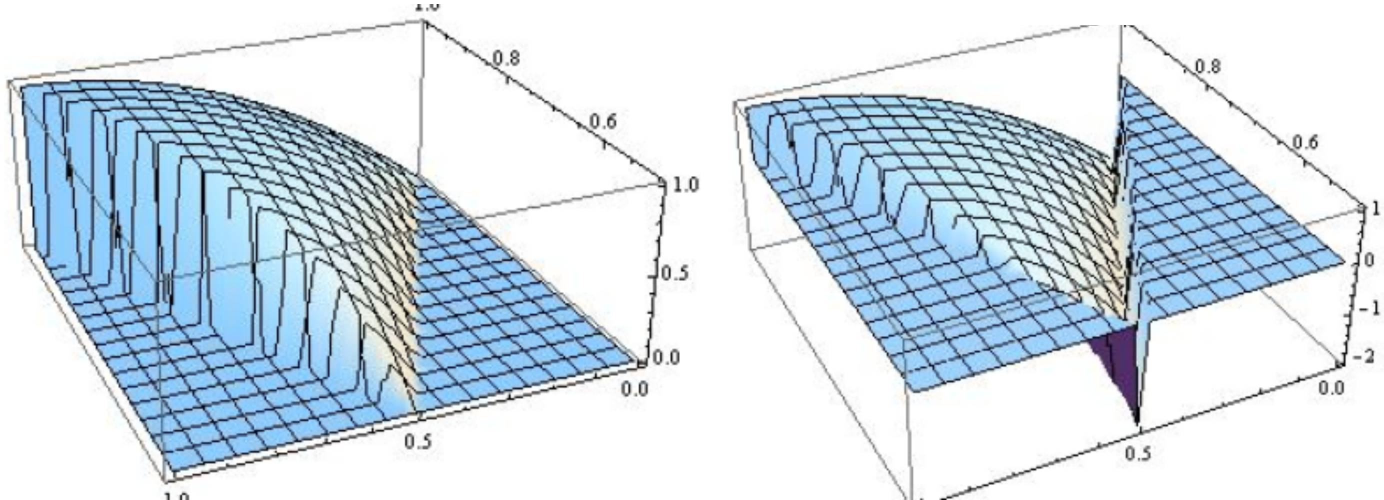
Non-Gaussianity

- Two parameters

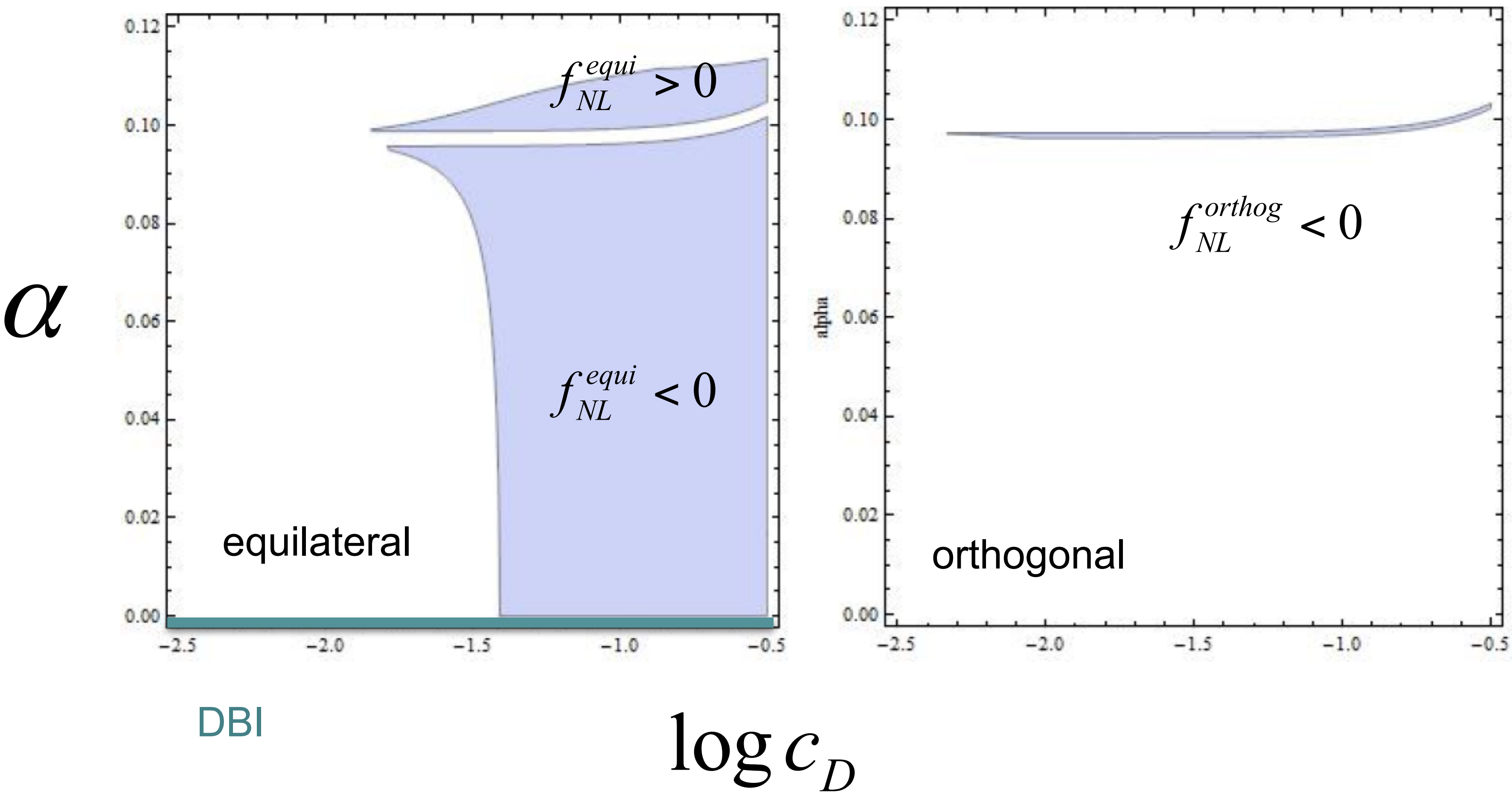
$$c_D^2 \equiv 1 - f\dot{\sigma}^2 \qquad \alpha = \frac{fH^2M^2}{c_D^2 h^{1/2}}$$

Mostly it gives equilateral NG but for some choice of parameters, there appears orthogonal type NG

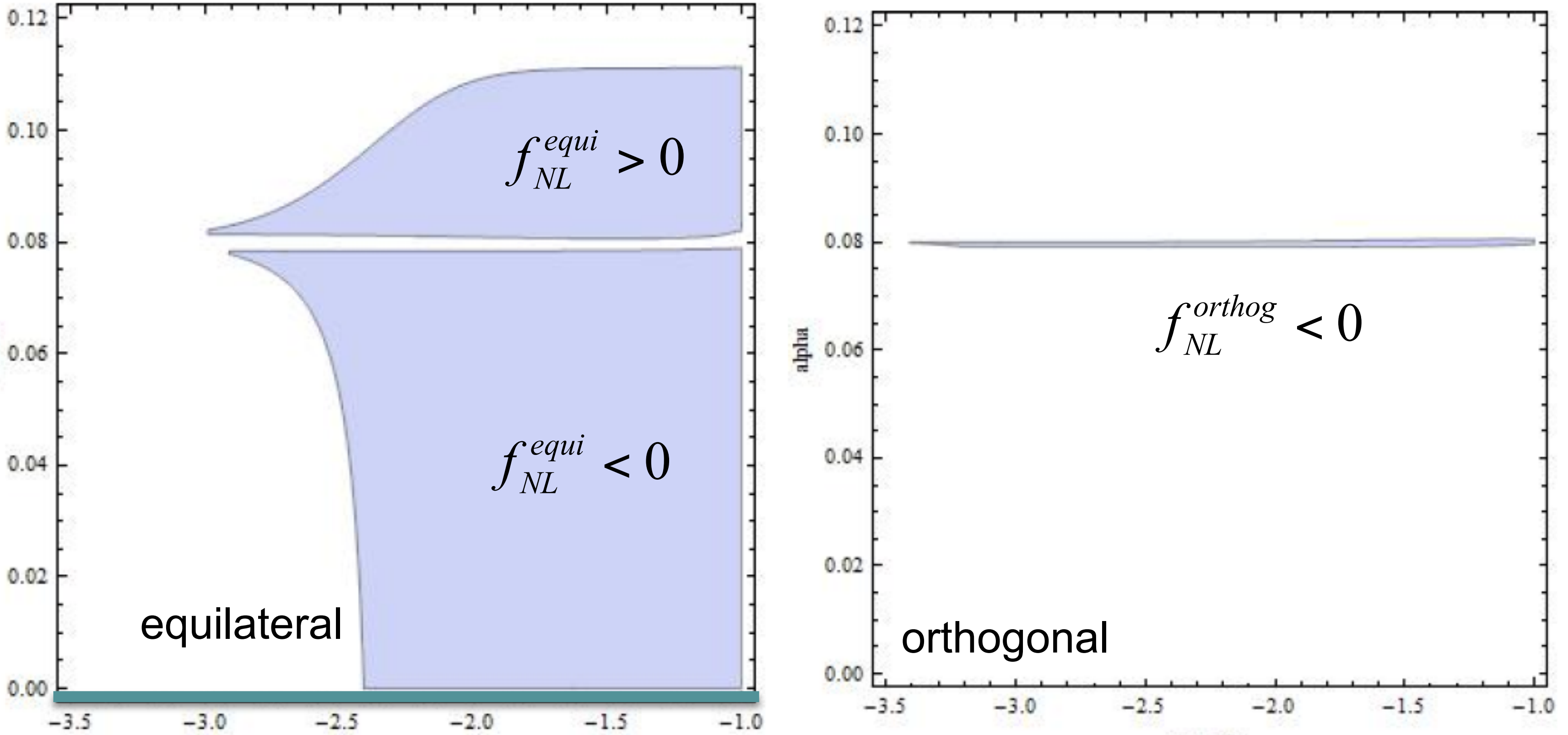
$$-214 < f_{\rm NL}^{equil} < 266 \qquad -410 < f_{\rm NL}^{orthog} < 6$$



Single field $T_{RS} = 0$



Multi field $T_{RS} = 10$



α

DBI

$\log c_D$

Conclusion

- Multi-field DBI galileon

  a probe brane action with induced gravity

- Induced gravity may break the energy condition

  need to check the ghost condition on non-slow roll inflation background

- Slow-roll inflation can be realised if $c_D f V >> 1, \quad \frac{M^2}{M_p^2 c_D^3 h^{1/2}} << 1$

- Cosmological perturbations are controlled by two parmaeters $c_D^2 \equiv 1 - f\dot{\sigma}^2 \quad \alpha = \frac{fH^2M^2}{c_D^2 h^{1/2}}$

  constraints on these parameters are obtained

  It is possible to create orthogonal type NG and positive f_{NL}^{equi}